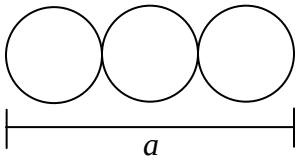
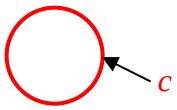
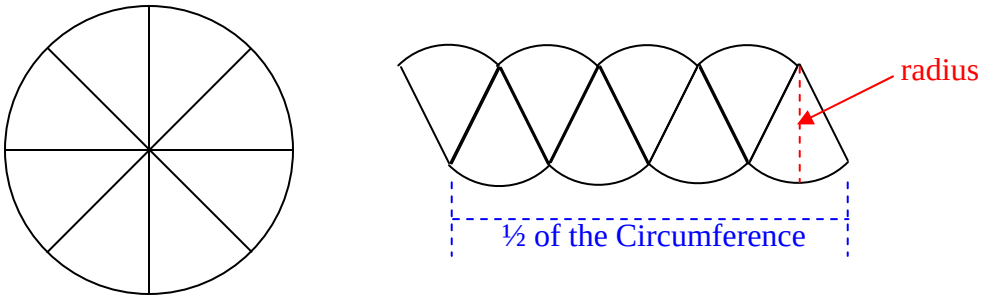
Math by Design
Lesson Plan: Circumference and Area of a Circle

National Standard: **Measurement**: Apply appropriate techniques, tools, and formulas…/develop and use formulas to determine…the area of…circles
MD Standard 3: Knowledge of Measurement
MD Topic C: Applications in Measurement
MD Indicator 1: Estimate and apply measurement formulas
MD Objective a: Estimate and determine the circumference or area of a circle
Materials and/or Set Up: Three each of several different sizes of lids, measuring tape and/or string and rulers
Relevant Vocabulary: Circle, center, radius, diameter, circumference, area
Note to Teacher: This lesson is designed to be used in conjunction with the online interactive activity at http://mathbydesign.thinkport.org.
Suggested Activities: ▪ Introduce the vocabulary associated with circles including circle, center, radius, diameter, and circumference. Provide practice with identifying them. ▪ Present the students with three lids from containers such as jars or coffee cans. Ask the following question: Which is longer, the total length (*a*) of the three lids when positioned side by side, or the distance around one of the lids? See the example below.       ▪ After the students have answered the question, demonstrate by using a measuring tape or a string that the circumference is longer than the total length. If necessary, repeat the procedure with a different set of lids and show that the result is still the same. ▪ If students have not previously had experience determining the value of pi, you may want to record the actual lengths and use them to show that the ratio of the circumference to the diameter is always a little more than 3. Introduce or review that this number is called pi and is represented by the Greek letter, π. ▪ Since the ratio of the circumference to the diameter is pi, lead the students to see that the circumference of a circle can be calculated (*or estimated to a very good approximation*) by multiplying the diameter by pi. Provide the students with practice in finding the circumference and be sure that they understand the difference between the exact answer, in terms of pi, and the approximation with a rounded or truncated decimal answer. ▪ Since students have already learned how to determine the area of a rectangle or

parallelogram, you may use these concepts to develop the formula for finding the area of a circle. Cut the circular region into triangular shapes (like pieces of pizza or pie) and rearrange them to make a parallelogram (with sort of wavy sides) as shown below. Note that the smaller the central angle used for the pieces, the "straighter" the sides of the parallelogram become.



- Ask students to recall the formula for finding the area of a parallelogram ($A = bh$) and then determine those measurements in terms of the measures that apply to the circle. The base of the parallelogram is half of the circumference of the circle. The height of the parallelogram is the radius of the circle. Substitute those measures into the area formula and simplify to produce the formula for finding the area of the circle as shown below.

$$\text{Area of Circle} = \text{Area of Parallelogram}$$
$$\text{Area of Circle} = bh$$
$$\text{Area of Circle} = \left(\frac{1}{2}C\right)(r)$$
$$\text{Area of Circle} = \frac{1}{2}(2\pi r)(r)$$
$$\text{Area of Circle} = (\pi r)(r)$$
$$\text{Area of Circle} = \pi r^2$$

- Provide practice in finding the areas of circular regions.

Differentiation Suggestions:

- Use the illuminations tool to further explore the relationships among the radius, diameter, circumference, and area of a circle: http://illuminations.nctm.org/ActivityDetail.aspx?id=116

Assessment:

- The radius of a circle is 7 cm. Find its circumference and its area.
  Answer: $C = 14\pi$ or about 43.9 cm. $A = 49\pi$ or about 153.9 cm^2.

Follow Up:

- Provide examples of semicircles and calculate their perimeters and areas.

