Math by Design
Lesson Plan: Pythagorean Theorem

National Standard: Geometry: Analyze characteristics…and develop mathematical arguments about geometric relationships/create and critique inductive and deductive arguments concerning geometric ideas and relationships, such as…the Pythagorean relationship	
MD Standard 2: Knowledge of Geometry	
MD Topic A: Plane Geometric Figures	
MD Indicator 2: Analyze geometric relationships	
MD Objective b: Apply right angle concepts to solve real-world problems	

Materials and/or Set Up: Graph paper, ruler, calculator

Relevant Vocabulary:
Right triangle, legs, hypotenuse, Pythagorean Theorem, square, square root, irrational number

Note to Teacher: This lesson is designed to be used in conjunction with the online interactive activity at http://mathbydesign.thinkport.org.

Suggested Activities:

- Arrange students into groups of three. Distribute graph paper to each student. Provide each student with the lengths of the legs of a right triangle that should be drawn in the middle of his/her graph paper. Use Pythagorean triples such as 3, 4, 5 and 5, 12, 13, and 6, 8, 10 so that the squares on the sides will fit on the graph paper and side lengths are whole numbers.

- Tell students to draw a square outside the triangle using one leg of the triangle as one side of the square. Direct them to repeat for the other leg of the triangle and finally for the hypotenuse. Ask students to determine the area of each of the squares. For the square using the hypotenuse, they may cut out squares from the grid and arrange them to cover the area. Another alternative is to use tracing paper or a transparent sheet of plastic to trace the square and move it to a position in which the grid squares can be used to determine the area.

- Direct the students to compare the areas of the squares on the legs of the triangle with the area of the square on the hypotenuse. Facilitate a discussion that leads to the conclusion: the area of the square on the hypotenuse of the right triangle is equal to the sum of the areas of the squares on the legs. Ask the students if anyone knows the name of this famous theorem from geometry. Provide a brief history of Pythagoras and introduce the name, Pythagorean Theorem.

- Help students to make the transition from the geometric figures to the words to the numbers and finally to the numerical statements as shown below:

Area of Square on Hypotenuse = Sum of Areas of Squares on Legs

Area of blue square = Area of red square + Area of purple square

$$25 = 9 + 16$$

$$5^2 = 3^2 + 4^2$$

- Provide the following example and ask the students to show the resulting Pythagorean statement that would apply to this triangle: The legs of a right triangle have lengths of 8 and 15 and the length of the hypotenuse is 17.

  Area of Square on Hypotenuse = Sum of Areas of Squares on Legs

  Answer:

  $$289 = 64 + 225$$

  $$17^2 = 8^2 + 15^2$$

- Pose the following question: Suppose you have a right triangle and you know that the lengths of the legs are 7 and 24 units. How could you use the Pythagorean Theorem to find the length of the hypotenuse? Ask a student to demonstrate and explain the process.

  Area of Square on Hypotenuse = Sum of Areas of Squares on Legs

  Answer:

  $$c^2 = 7^2 + 24^2$$

  $$c^2 = 49 + 576$$

  $$c^2 = 625$$

  $$\sqrt{c^2} = \sqrt{625}$$

  $$c = 25$$

- Ask students if they think that every right triangle has side lengths represented by whole numbers. Provide the following example of a triangle for which the length of the hypotenuse is an irrational number. Use the Pythagorean Theorem to find the length. Inform students about how they should provide irrational answers (*as exact answers using radicals, simplified radicals or rounded to a specific place value*.)

  Area of Square on Hypotenuse = Sum of Areas of Squares on Legs

  Answer:

  $$c^2 = 8^2 + 12^2$$

  $$c^2 = 64 + 144$$

  $$c^2 = 208$$

  $$\sqrt{c^2} = \sqrt{208}$$

  $$c = \sqrt{208} \text{ or } 4\sqrt{13}$$

  $$\text{or } c \approx 14.42$$

- Provide practice with using the Pythagorean Theorem to find the length of the hypotenuse.

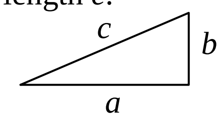
- When students are ready, change the problem so that they are given the length of the hypotenuse and one leg of the triangle and asked to find the length of the other leg. Provide practice with this type of problem. Also provide practice in which students must decide whether they are finding a length of a leg or of the hypotenuse.

Differentiation Suggestions:

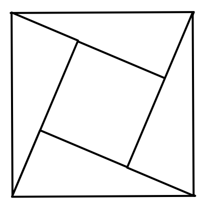
- Students may combine areas from the squares on the leg to cover the area on the hypotenuse using the activity at: http://www.ies.co.jp/math/java/geo/pythasvn/pythasvn.html

- For students who understand how to square a binomial, use the following proof of the Pythagorean Theorem:

  Begin with a right triangle in which the longer leg has length *a*, the shorter leg has length *b*, and the hypotenuse has length *c*.

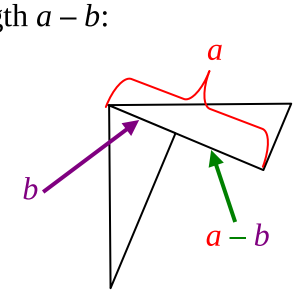


  Make four copies of the triangle and arrange them as shown here:



  Note that the resulting figure is a square with sides of length c. You may need to explain that you know the angles are right angles because they are formed by the two acute angles from the right triangles. The area of this resulting square is c^2.

  Then point out that the small figure in the center of the large square is also a square with sides of length $a – b$:



  Consequently the total area of the large square can also be calculated by adding the areas of the four triangles (each is ½ *ab*) to the area of the small square (which is the square of the binomial $a – b$.)

  The result is:

Area of Large Square = Area of Small Square + Areas of Triangles

$$c^2 = (a-b)^2 + 4\left(\frac{1}{2}ab\right)$$

$$c^2 = (a^2 - 2ab + b^2) + (2ab)$$

$$c^2 = a^2 + b^2$$

Assessment:

- The size of a TV screen is usually given as the length of its diagonal. If the diagonal measure is 42 inches and the width of the screen is about 36.6 inches, what is its approximate height?
  Answer: The height is about 20.6 inches.

Follow Up:

- Ask students to write a report about the Pythagorean Theorem and how it was used in ancient times.

- Students may also want to research and report on some of the proofs of the theorem.

